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Designing a Hypothetical Learning Trajectory with Student Worksheets: Fostering Collaboration and Innovation in Probability Learning

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Abstract

This study aims to develop a Hypothetical Learning Trajectory (HLT) based on student worksheets for teaching probability as an innovative approach to enhance students' conceptual understanding. The development is driven by the need for instructional design that connects abstract mathematical concepts to real-life contexts. The research adopts a design-based research (DBR) approach through an in-depth case study of three eleventh-grade students. Data were collected through LKPD tasks, observations, semi-structured interviews, and diagnostic tests. Findings show that the HLT-based student worksheet effectively supported students' progressive understanding of probability, fostering intuitive exploration and formal application. Students demonstrated active engagement, cognitive growth, and positive attitudes toward learning. The study concludes that implementing HLT through LKPD is a valid and promising strategy to improve the quality of mathematics instruction, particularly in the topic of probability.

Keywords: *Hypothetical Learning Trajectory; Student Worksheet (LKPD); Probability; Design-Based Research; Conceptual* Understanding;

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INTRODUCTION

Mathematics is a core discipline that supports logical reasoning, problem-solving, and decision-making in various real-life contexts. One essential topic within mathematics is probability, which deals with uncertainty and the likelihood of events. Although probability has broad applications from games and risk analysis to statistics and finance many high school students struggle to understand it due to its abstract nature and lack of connection to tangible experiences (Nova et al., 2022a; Nuraida & Putri, 2019).

Conventional pedagogical methods frequently prioritize rote memorization of formulas above conceptual comprehension, so constraining students' capacity to apply probability effectively (Juana et al., 2022a). Current research has examined context-based learning models to enhance reasoning and participation. It has been demonstrated that cultural items, like traditional cuisine or Palembang's Songket textile, can assist students in developing an intuitive comprehension of mathematical ideas in situations they are familiar with (Nuraida & Putri, 2019; Sari & Putri, 2021).

The researcher took three students with varying levels of mathematical ability to be fully involved in the initial practical exercises. This study obtained results on student worksheets due to differences in students' reasoning abilities in converting real experiences into abstract representations, this shows that the need for educational resources by combining structured teaching with contextual exploration greatly influences the student learning process.

HLT helps students move from informal to formal mathematical thinking by coordinating education with their developmental stages when combined with Realistic Mathematics Education (RME) (Adha et al., 2024). Though HLT has been used in the classroom in a number of studies, the majority depend on active instructor participation. Few studies have looked at how well-designed student worksheets, especially when it comes to probability, can independently support and assess students' learning paths. Given that LKPDs with real-life contexts and exploratory activities have the ability to promote autonomous reasoning and conceptual understanding, this is a crucial gap (Nova et al., 2022a; Rahayu et al., 2022; Sahara et al., 2024).

By creating and evaluating a student worksheet for probability instruction based on the HLT paradigm, our study fills that knowledge gap. Without the need for direct teacher assistance, the LKPD aims to lead students through the whole learning process, from intuitive discovery to symbolic representation and practical application.

Using a design-based research technique, the study investigates how three eleventh-grade students with varying degrees of mathematical proficiency interact with the LKPD and how their thinking evolves as they complete various tasks. By combining problem-solving, practical application, and cultural context into a unified HLT-based design, this project advances mathematics education conceptually and practically. It demonstrates how student worksheets can serve as independent resources and instructional tools in addition to being used to plan and assess students' conceptual growth in probability.

Additionally, as educational technology has advanced, more digital resources like augmented reality, interactive movies, and simulations are being used in math classes. Prior research indicates that augmented reality-based instructional strategies can boost student interest and ease the shift from tangible to intangible ideas (Andzin et al., 2024). As a result, integrating aspects of local technology and culture makes it possible to create HLT-based LKPD for probability.

METHODS

In the section, please explain clearly how to conduct your research in order to: (1) enable readers to evaluate the work performed and (2) permit others to replicate the research. The author must describe exactly what he/she did: what and how experiments were run, what, how much, how often, where, when, and why equipment and materials were used. The main consideration is to ensure that enough detail is provided to verify the findings and to enable the replication of the research.

This study is development research that adopts the Design-Based Research (DBR) approach. This approach was chosen because it aligns with the objective of the study, which is to develop and evaluate the initial effectiveness of a student worksheet based on the Hypothetical Learning Trajectory (HLT) framework in the topic of probability. DBR is designed to bridge the gap between theory and educational practice through systematic processes of design, implementation, and reflection in real learning environments.

DBR also provides flexibility for researchers to develop solutions to complex learning problems while contributing contextually grounded and applicable theories. In this context, HLT served as the theoretical framework for designing students' learning trajectories in understanding probability, from informal to formal stages, through step-by-step and context-based activities (Mouli et al., 2023; Nilsson, 2023; Utari et al., 2024)

This study was conducted in March 2025 at a senior high school in Ngawi city. The research subjects consisted of three eleventh-grade students selected purposively based on mathematical ability levels (high, medium, and low), as recommended by the mathematics teacher. The implementation was carried out individually, without teacher intervention, to examine how students interacted independently with the developed LKPD.

The DBR stages applied in this study follow the framework by Gravemeijer and Cobb (2006), which includes: (1) preliminary analysis and theoretical review, (2) initial design development, (3) implementation and data collection, and (4) reflection on the design.

In the first phase, the researcher conducted a preliminary study by analyzing curriculum documents, conducting informal interviews with a mathematics teacher, and reviewing literature related to students' difficulties in understanding probability. The analysis showed that the teaching of probability is often procedural and lacks connection with students' real-life experiences (Nova et al., 2022b). Furthermore, the teacher stated that no existing LKPD adequately facilitated students' conceptual thinking in a step-by-step and contextualized manner.

The second phase was the design of a prototype instructional worksheet based on HLT. The researcher developed a hypothetical learning trajectory consisting of three main components: learning objectives, task sequences, and predicted student responses (Adha et al., 2024; Rahayu et al., 2022). The iceberg learning trajectory was designed to help students develop an understanding of probability through exploratory activities. The LKPD was built around familiar contexts such as spinner and dice games, progressing toward semi-formal representations like frequency tables and tree diagrams, and eventually symbolic mathematical expressions of probability.

The initial LKPD design was then validated by two experts: one mathematics education lecturer and one high school mathematics teacher. The validation used an evaluation sheet based on content, language, and feasibility criteria. Based on the feedback from the validators, the researcher revised the activity flow and clarified certain task instructions.

The third phase involved limited implementation of the LKPD with three students on an individual basis. The activity was conducted in two separate sessions, each lasting 90 minutes. During implementation, the researcher observed student behavior, recorded the problem-solving strategies used, and collected data through: (1) students' written work on the LKPD and (2) semi-structured interviews conducted after the activity. Observations were made without intervention or guidance, to ensure that the data reflected the students' independent understanding of the material.

The fourth phase was reflection on the design. The researcher conducted a comparative analysis between the students' actual learning trajectories and those predicted in the HLT. Data were analyzed using a descriptive qualitative approach to identify: (1) the alignment between student responses and the HLT, (2) the emergence of misconceptions, and (3) the effectiveness of each activity in facilitating the transition from concrete contexts to formal mathematical abstraction (Rahayu et al., 2022; Sahara et al., 2024).

To enhance the validity of the findings, triangulation was applied by comparing three sources of data: student worksheet responses, transcripts from post-activity interviews, and observation notes. This allowed for a more complete understanding of how students used the learning trajectories presented in the worksheet to construct their probability concepts.

FINDING AND DISCUSSIONS

The overall efficacy of the Descriptive Approach Learning Activity Plays is largely determined by the structured elements of the reflection and organization, which are based on students' emotional reactions during the classroom learning process. The rise in deeper engagement has a significant impact on the degree of involvement and seriousness with which pupils approach the activity being used. Emotional attachment frequently prompts more in-depth investigation, indicating that educational resources have to be created to balance the cognitive and affective facets of student learning.

Furthermore, it has been demonstrated that include open-ended questions in student worksheets greatly enhances their ability to think critically. These questions are meant to help students express their ideas, provide logical explanations for their conclusions, and communicate their implicit understanding to the teacher and to themselves. As a result, researchers can more precisely identify student errors, knowledge gaps, and variations in concept comprehension. This approach greatly supports the development of mathematical communication skills, which are occasionally overlooked in high school math classes. Students' understanding improves and a classroom culture that values discussion and introspection is established as they are encouraged to elaborate and validate their ideas.

The students had a very different understanding of probability after the lesson. Many people who had previously shown doubt, hesitation, or unease began to show noticeably more confidence and readiness to take on probability-related problems. This modification shows how students' views of their own abilities as competent mathematicians and problem solvers may benefit from the Hypothetical Learning Trajectory (HLT) method of instruction. Because of the exploratory and iterative nature of the student worksheet activities, which allow for inquiry, mistake, and development, students can engage with challenging and complex subjects in a supportive learning environment.

The transition from observations to mathematical concepts may be facilitated by visual aids such as pie charts, bar graphs, and even interactive animations. An increasing amount of research indicates that visual learning techniques and multimedia technologies can improve student understanding and engagement, especially in problem-based learning settings (Aripin et al., 2025).

All of the study's results provide strong support for the use of Hypothetical Learning Trajectories as a comprehensive teaching framework that fosters the development of emotional and metacognitive intelligence in addition to conceptual comprehension. The findings also demonstrate the importance of student-centered instructional design in mathematics education, especially when creating resources that actively involve students in relevant and fulfilling learning activities.

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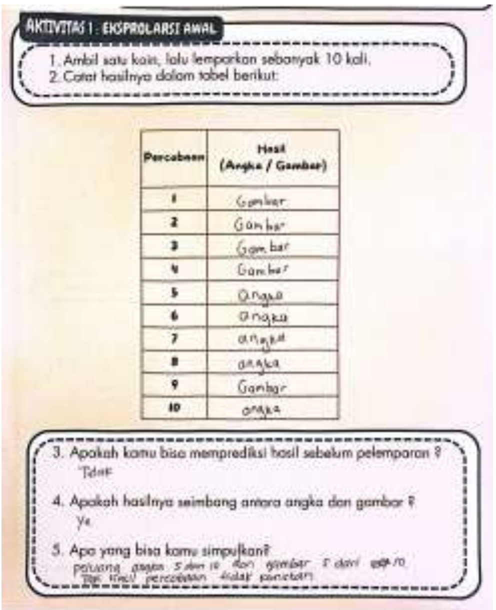
Initial Design and Learning Objectives

The student worksheets are organized in a systematic and planned way to assist students in building a solid conceptual foundation in probability. Establishing learning objectives is consistent with the conclusion of Phase F of the Merdeka Curriculum, which highlights the importance of inferring conclusions from theoretical reasoning and current knowledge when calculating the likelihood of simple and compound events. To guarantee consistency and relevance, five learning activities have been created based on the Hypothetical Learning Trajectory (HLT) principles. Curriculum analysis and information gathered from math teachers' interviews at SMA Negeri 1 Kedunggalar served as the basis for the activities.

The RME framework and inductive reasoning are both used in this development, which adheres to the procedure described by Sari and Putri (Sari & Putri, 2021). The three main components of the HLT are a set of learning exercises designed to improve conceptual knowledge, clear learning objectives, and an evaluation of students' thought and response patterns. By starting with concrete assessments from everyday life, such as throwing dice and money, this teaching approach uses the "iceberg" technique. These exercises lay the groundwork for nearly formal tools such as frequency tables, which in turn allow probability to be expressed symbolically through fractions and ratios.

a. Activity 1: Coin Toss Experiment



AKTIVITAS 1 : EKSPROLARSI AWAL

1. Ambil satu koin, lalu lemparkan sebanyak 10 kali.
2. Catat hasilnya dalam tabel berikut:

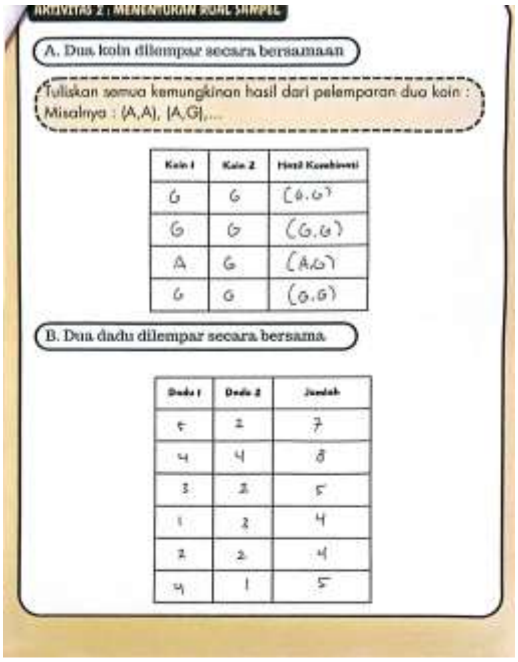
Percobaan	Hasil (Angka / Gambar)
1	Gambar
2	Gambar
3	Gambar
4	Gambar
5	angka
6	angka
7	angka
8	angka
9	Gambar
10	angka

3. Apakah kamu bisa memprediksi hasil sebelum pelemparan ?
Tidak
4. Apakah hasilnya seimbang antara angka dan gambar ?
Ya
5. Apa yang bisa kamu simpulkan?
peluang angka 5 dari 10 dan gambar 5 dari [illegible] 10 [illegible] hasil percobaan tidak [illegible]

Figure 1. Subject 1: Activity 1

Students report the results of ten coin flips in a frequency table. They consider issues like "Are the results balanced?" "Could this result have been anticipated?"

b. Activity 2: Compound Experiments (Two Coins and Two Dice)



A. Dua koin dilempar secara bersamaan

Tuliskan semua kemungkinan hasil dari pelemparan dua koin :
Misalnya : (A,A), (A,G),...

Koin 1	Koin 2	Hasil Kombinasi
G	G	(G.G)
G	G	(G.G)
A	G	(A.G)
G	G	(G.G)

B. Dua dadu dilempar secara bersama

Dadu 1	Dadu 2	Jumlah
5	2	7
4	4	8
3	2	5
1	2	4
2	2	4
4	1	5

Figure 2. Subject 3: Activity 2

Students reinforce the connection between empirical and predicted results by tossing two coins and rolling two dice ten times, then comparing the observed results with theoretical probabilities.

c. Activity 3: Generalization and Reasoning

Figure 3. Subject 2: Activity 3

Students calculate theoretical probabilities based on their experiments, represent them as ratios or fractions, and attempt to make predictions in new situations.

d. Activity 4: Real-Life Applications

Figure 4. Subject 1: Activity 4

Students solve probability problems in contexts such as rock-paper-scissors and marble games, discussing how probability affects decision-making.

e. Activity 5: Critical Reasoning Task

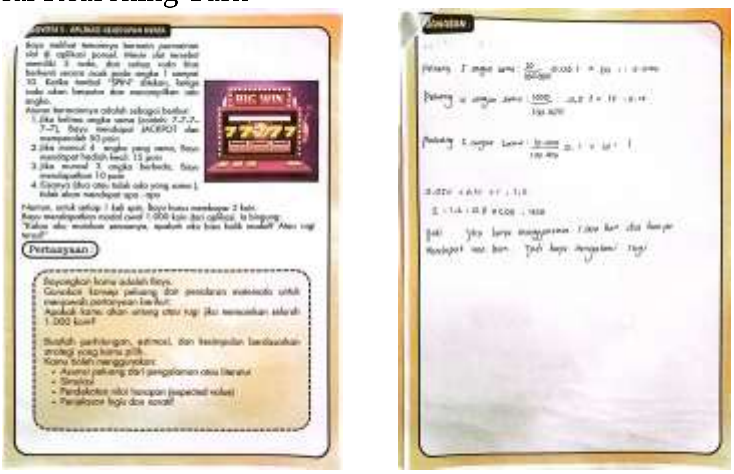


Figure 5 & 6. Subject 3: Activity 5

Students analyze a scenario involving slot-machine gambling. This task prompts critical thinking and risk awareness by examining unfair probabilities and discussing potential consequences.

Implementation and Student Responses

The LKPD was implemented with three eleventh-grade students of varying mathematical abilities (high, medium, and low) over two 90-minute sessions. Each student completed the worksheet independently, without teacher intervention. Data were collected through students' written work, direct observations, and post-activity semi-structured interviews.

Key findings include the following insights, which were observed consistently throughout the implementation phase and confirmed through triangulation of data sources (student work, interviews, and observation notes):

a. In Activities 1 and 2, all students were engaged. The high-ability student organized data quickly and identified patterns. The medium-level student followed the steps but lacked precision. The low-ability student recorded data but struggled with structure.

Figure 7. Subject 2 & 3: Throwing dice and coins

b. In Activity 3, the high-ability student accurately expressed probabilities as fractions and justified them. The medium-level student formed correct ratios but misunderstood their meaning. The low-ability student recognized which outcomes appeared more often but could not articulate this as a probability.

Figure 8. Subject 1: Do activity 3

c. Activity 4 (decision-making scenarios) generated enthusiasm from all students due to its familiar context. The task stimulated discussions on fairness and prediction.
d. Activity 5 (slot-machine scenario) prompted critical thinking. The high- and medium-level students concluded that the game was unfair. The low-level student was interested in the scenario but lacked the quantitative reasoning to evaluate its risks.

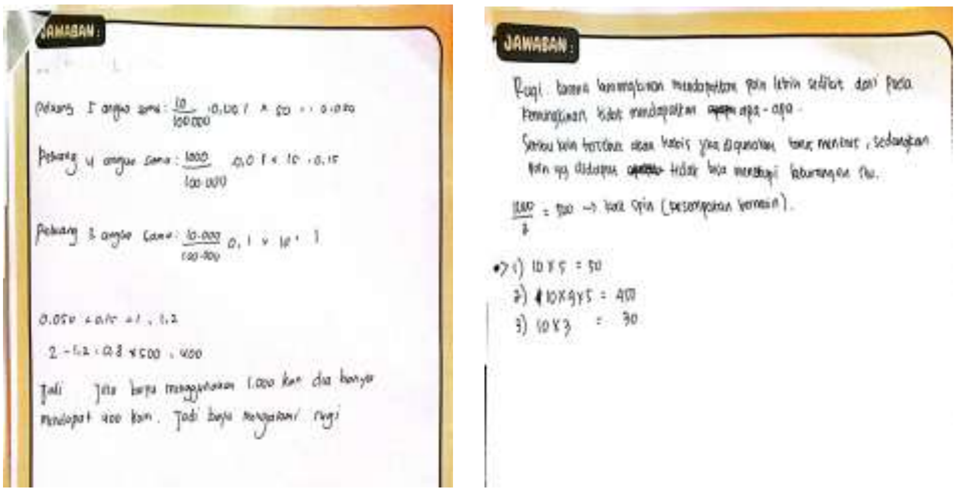


Figure 8 : answers of students with high ability and students with low ability

While this study used common contexts such as coin toss and spinner games, future iterations could explore more dynamic media and cultural integration as suggested (Nursyahidah et al., 2025) and supported by the instructional impact seen in the augmented reality-supported LKPD developed (Andzin et al., 2024).

In order to provide students with a well-structured and coherent pathway toward developing a robust conceptual understanding of probability, a Hypothetical Learning Trajectory (HLT) was constructed with foundational support from the principles of Realistic Mathematics Education (RME) and inductive reasoning. This learning trajectory was deliberately designed to reflect the natural progression of student thinking, starting from intuitive experiences and gradually advancing toward more formalized mathematical reasoning.

The HLT itself was organized around three fundamental components: clearly defined learning objectives, a carefully sequenced set of instructional activities, and anticipated patterns of student thought. Each of these elements played an essential role in supporting students' cognitive development. By anticipating how students might initially interpret and approach the material, the HLT made it possible to design instructional steps that would both challenge and support learners as they moved through increasingly complex levels of understanding.

Furthermore, each phase of the learning process was purposefully aligned with these components to ensure that students were being gradually and intentionally guided. The structure was not only meant to foster comprehension, but also to promote reflection, reasoning, and connection-making. The following table outlines the detailed design of this HLT, demonstrating how every learning activity was strategically developed to facilitate students' progressive understanding of probability concepts.

Table 1. Structure of the developed HLT for teaching probability.

No	Stage	Learning Goal	Learning Activity	Predicted Student Response
1.	Concrete Exploration	Students recognize that the outcomes of probability experiments are random and not always balanced.	Toss a coin or roll a die 10 times and record each result.	Students actively record data. They notice variation in outcomes and begin to grasp the idea of randomness.
2.	Semi-formal Representation	Students can organize experimental data in a table and compare frequencies.	Build a frequency table for two-coin or two-die experiments and compare empirical results with theoretical probabilities.	High- and medium-ability students create tables correctly; lower-ability students need guidance.
3.	Generalization & Symbolization	Students can express probabilities as fractions/ratios and link them to theoretical concepts.	Calculate probabilities from the collected data and write them as fractions.	High-ability students articulate probabilities accurately; others may misunderstand or misrepresent ratios.
4.	Real-life Application	Students identify and apply probability concepts in everyday contexts.	Solve contextual tasks (e.g., rock-paper-scissors, marble draws) and discuss decision-making based on probability.	All students engage enthusiastically; they start seeing how probability informs real decisions.
5.	Critical Reasoning	Students use probability to assess risk- laden situations and make rational decisions.	Analyze a slot-machine scenario: compute expected gains/losses and decide whether to play.	High- and medium-ability students conclude the game is unfair; lower-ability students show interest but struggle with quantitative reasoning.

Actual vs. Hypothetical Learning Trajectories

To evaluate how students' actual learning compared with the predicted HLT, their written work, interview responses, and observations were analyzed. A comparison table 2 summarizes the results.

Table 2. HLT Actual vs Predicted Comparison

No.	HLT Stage	Design Prediction	Actual Findings
1	Concrete Experiment (coin/dice)	Students record results and recognize distribution	All students active; low-ability student needed help recording
2	Semi-formal Representation	Students build tables and compare frequencies	High and medium students succeeded; low-level student fell behind
3	Generalization & Symbolization	Express probability in fractions	Only high-ability student stated probabilities accurately
4	Real-Life Applications	Analyze probability in real contexts	All students interested and engaged
5	Critical Reasoning (slot machine)	Conclude uneven chance in practice	High/medium students concluded risks of gambling

The data indicated that early phases of the learning trajectory (concrete exploration) were achieved by all students, confirming that real-world, game-based tasks successfully foster engagement (Nilsson, 2023). These initial activities served as a bridge between intuitive understanding and structured reasoning, creating a foundation for students to build upon in later stages. However, difficulties emerged in the transition to symbolic representation, particularly among lower-ability students. They struggled to move from contextual understanding to mathematical abstraction, suggesting the need for more scaffolding. For example, while some students were able to intuitively identify patterns in empirical data, they were unable to represent these patterns symbolically using appropriate mathematical language. Despite this, the final activity (critical reasoning) generated meaningful discussions even from students with medium-level ability, showing that real-life contexts can foster deeper thinking. This reinforces the importance of incorporating relevant, familiar situations that resonate with students' everyday experiences.

Design Reflection and Improvements

Despite their initial narrow focus, the student worksheets (LKPD) yielded several valuable insights that could be applied to future iterations of the instructional design. Even though the worksheets helped students gradually gain a deeper understanding of the concept of probability, some sections still need careful study. These improvements aim to meet the needs of a diverse class by accounting for various learning styles, cognitive processes, and academic readiness levels.

First of all, students who had trouble with abstraction had the most trouble, especially with assignments that required them to move from empirical data to theoretical interpretations. One of the main reasons for this issue was that students were not given clear instructions, which made it difficult for them to calculate theoretical probabilities or interpret the data they had gathered. Previous studies on technology-based learning in mathematics suggest that advanced technologies like augmented reality and dynamic modeling tools can improve students' conceptual understanding (Andzin et al., 2024).

The transition from observations to mathematical concepts may be facilitated by visual aids such as pie charts, bar graphs, and even interactive animations. An increasing amount of research indicates that visual learning techniques and multimedia technologies can improve student understanding and engagement, especially in problem-based learning settings (Aripin et al., 2025).

The worksheet's reflection section also seemed to be underutilized, in addition to this structural limitation. Many students completed them quickly and often gave flimsy, thoughtless answers. According to this pattern, metacognitive skills ought to be specifically taught throughout the educational process. Deeper thinking can be prompted by structured reflection questions such as "What patterns do you see in your results?" or "Can you relate this concept to your everyday experience?" This method will not only simplify learning for children, but it will also make learning easier for them.

In addition to helping students remember the material, this method will provide teachers with more accurate information to assess their students' understanding. Combining self-reflection with local knowledge may significantly improve comprehension and encourage student participation, per Nursyahidah's research (Nursyahidah et al., 2025). Additionally, students will have more opportunities to monitor their cognitive development over time and address any misunderstandings if this reflection is carried out regularly throughout the learning process rather than just at the end.

Collectively, these design ideas demonstrate how important it is to give students worksheets that are not only technically sound but also visually appealing, suitable for the student's culture, and easy to understand. These findings are in line with past studies that advocate for integrating local culture into mathematics education and exploratory learning frameworks (Nilsson, 2023; Rahayu et al., 2022; Utari et al., 2024). Additionally, the suggested improvements are in line with current industry trends that place a higher value on relevant and student-centered learning environments than rigid and generic teaching strategies (Sari & Putri, 2021). Worksheet design can better meet students' learning needs and create a more equitable learning environment by fusing pedagogical theory with real-world classroom experience.

In the future, a more thorough assessment of this teaching tool across various academic contexts and student demographics may shed light on its generalizability and scalability. Researchers will be able to evaluate the tool's adaptability through additional trials in different educational contexts with larger and more diverse student groups. In order to assess deeper learning, longitudinal studies should also look at whether learning gains are maintained over time and whether students can apply probabilistic reasoning techniques in unfamiliar situations (Adha et al., 2024; Hadila et al., 2020).

More significantly, the diversity of learning styles, comprehension velocities, and multiple intelligences seen in today's classrooms emphasizes the value of adaptable and flexible educational materials. It is possible to enhance inclusion without compromising academic integrity by developing learner worksheets that support individualized instruction by using tiered assignments or providing activity alternatives based on each student's aptitude. Since students usually have preconceived notions that need to be carefully addressed and cognitively altered, this method is especially important when teaching probability. According to the HLT concept, learner worksheets can help students at different levels make meaningful progress by offering a variety of entry points and active learning strategies.

In a nutshell, HLT-based Learner Worksheets' strongest point is their ability to guide students through an adaptable yet structured curriculum that encourages emotional engagement and understanding of mathematical concepts. By adjusting the teaching strategies and learning process to each student's unique needs, this tool helps students become more motivated and less anxious about challenging subjects like probability. When students feel that the material being taught suits their learning style and pace, they are more likely to think of themselves as proficient mathematical thinkers. This is an essential part of successful and equitable math education (Nilsson, 2023).

Theoretical and Practical Implications

The researcher took three students with varying levels of mathematical ability to be fully involved in the initial practical exercises. This study obtained results on student worksheets due to differences in students' reasoning abilities in converting real experiences into abstract representations, this shows that the need for educational resources by combining structured teaching with contextual exploration greatly influences the student learning process.

Through direct student interaction and iterative refinement of Student Worksheets (LKPD) made possible by the development of LKPD based on the DBR method, researchers and educators can jointly develop learning media used during the learning process with pedagogically sound materials that address real-world classroom dynamics while being firmly rooted in educational theory.

This study specifically adds to the reference by showing how to apply HLT in practice without solely relying on teacher-led instruction by using self-learning resources, especially learner worksheets (LKPD). In order to improve mathematical comprehension, (Adel, 2020)underlined the significance of learning advancements in line with HLT, particularly when coupled with Realistic Mathematics Education (RME) concepts. This is consistent with the Realistic Mathematics Education (RME) tenets and Adel's findings.

Hadila, Sukirwan, and Alamsyah (Hadila et al., 2020) provided additional support for this teaching strategy by illustrating how contextualized exploration utilizing RME-based teaching strategies may improve students' understanding of planar geometry. This is why Fitri (Fitri, 2022)showed how incorporating well-designed visual elements into learning activities, such as the Plusminus project, could improve students' interest and spatial reasoning skills, especially when they are studying the Pythagorean theorem. These findings are in line with the corpus of existing research on contextual learning resources and how they can increase accessibility and interest in abstract mathematics.

Future research may look at different facets of using the learner workbooks based on the findings. One strategy could be to look at how students use the learner worksheets in cooperative groups and how teachers use them in a real classroom. Examining how frequently HLT is applied in other abstract mathematical domains, like combinatorics or statistical inference, would be an intriguing strategy. Such studies could evaluate the effectiveness of a thoughtful but adaptable design approach in a variety of mathematical fields.

This study demonstrates how effective the HLT framework is at organizing learning for complex mathematical concepts. HLT provides teachers with a solid basis for developing contextualized, continuous learning that can predict students' thought processes and adapt activities based on stages of cognitive development. Previous use of HLTs in more complex materials, like area under a curve (Fauzan, 2024)and hypothesis testing (Nilsson, 2023), shows how flexible and scalable these HLTs are for different grade levels and subjects..

The results of the study show that when students are given the opportunity to explore mathematical concepts such as conducting direct experiments or questions that are designed according to the real context that students have experienced and with the support of visual learning media, cultural content, and the context of students' daily lives, the most effective learning experience can be realized. Students will connect understanding through the iterative thinking patterns of exploration or experimentation, representation, and reflection that are essential to HLT and RME, which are based on observed learning patterns.

As a result, the Student Worksheets prepared in this study serve as a teacher intermediary in teaching small group learning and independent learning, especially in educational institutions that support inquiry-based learning or differentiated learning. In this study, although only three students were involved, the various answers they gave showed the natural adaptability of the design.

While this study used common contexts such as coin toss and spinner games, future iterations could explore more dynamic media and cultural integration as suggested (Nursyahidah et al., 2025) and supported by the instructional impact seen in the augmented reality-supported LKPD developed (Andzin et al., 2024).

CONCLUSION

The results of this study aims to develop a Hypothetical Learning Trajectory (HLT) based on student worksheets for teaching probability as an innovative approach to enhance students' conceptual understanding. The development is driven by the need for instructional design that connects abstract mathematical concepts to real-life contexts.This is achieved through the Design-Based Research (DBR) approach, which includes the design, implementation, and evaluation phases repeatedly.

The researcher took three students with varying levels of mathematical ability to be fully involved in the initial practical exercises. This study obtained results on student worksheets due to differences in students' reasoning abilities in converting real experiences into abstract representations, this shows that the need for educational resources by combining structured teaching with contextual exploration greatly influences the student learning process.

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Through direct student interaction and iterative refinement of Student Worksheets (LKPD) made possible by the development of LKPD based on the DBR method, researchers and educators can jointly develop learning media used during the learning process with pedagogically sound materials that address real-world classroom dynamics while being firmly rooted in educational theory.

The benefits of developing this Student Worksheet (LKPD) in the teaching and learning process in the classroom carried out by students and teachers can produce larger and more diverse situations, especially in situations that encourage collaborative learning, which is very important for future implementation.

The effect of learning materials that are in accordance with the HLT concept can be significantly enhanced by including techniques such as exploration activities carried out by students directly and formative evaluations based on students' written explanations, and rapid instructor feedback. Furthermore, if this material is routinely included in a longer curriculum or in several learning cycles, researchers can further explain how students' probabilistic thinking changes over time (Mouli et al., 2023; Nilsson, 2023).

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